Exam Nonlinear Optics

Wednesday, April 4, 2018, 14.00-17.00 (room 5114.0004)

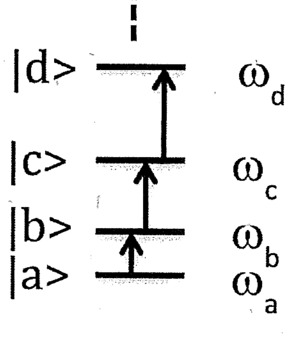
Give your name on each sheet.
On the first sheet, also give your student number
and the total number of sheets turned in.
Score = total points/2.9
Success!

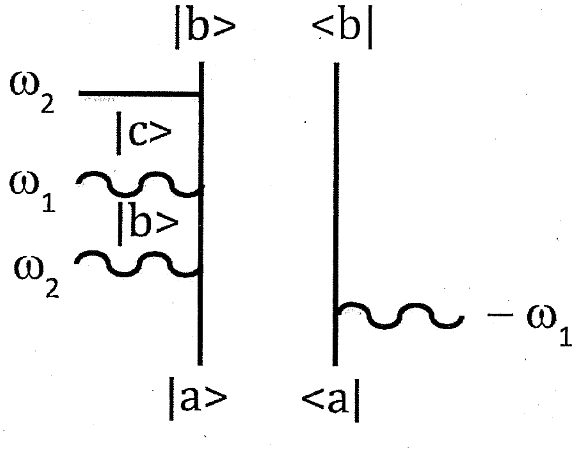
Problem 1 (7 points, distributed according to (a,b,c,d) = (1, 2, 2, 2))
Consider a material consisting of one-dimensional classical damped anharmonic oscillators with a quadratic force term $-max^2$ as nonlinearity. The density of oscillators is denoted N. Furthermore, consider an incident monochromatic (c.w.) light beam, i.e., $E(t)=E(\omega)\exp(-i\omega t)$ + c.c. In the problems below, always explain the quantities you introduce.

a) Give the equation of motion for the oscillators.
b) Derive the susceptibility $\chi^{(1)}(\omega)$. From this, give an expression for the linear absorption coefficient $\alpha(\omega)$ near resonance and show that this has a Lorentzian lineshape.
c) Derive the susceptibility $\chi^{(2)}(2\omega,\omega,\omega)$. What process does this susceptibility describe?
d) Derive the susceptibility $\chi^{(3)}(3\omega,\omega,\omega,\omega)$.

Problem 2 (8 points, distributed according to (a,b,c) = (3, 3, 2))
Consider the molecular energy level diagram below, which consists of a ladder of energy levels. The only nonzero dipole matrix elements exist between neighboring levels, so between |a> and |b>, between |b> and |c>, etc. All other dipole matrix elements are zero. The dephasing rate of the coherence |a><b| is given by γ_{ab}, and analogous for the other coherences. The molecule initially is in the ground state |a>.

a) Argue on the basis of the general properties of Feynman diagrams for this system what is the lowest order nonlinear response that is sensitive to the state |d>, in other words, what is the lowest n for which the state |d> plays a role in the nonlinear susceptibilities of order n?
b) Calculate the third-order diagram drawn next to the level picture, where two (real) c.w. fields are incident on the system, with positive frequencies ω_1 and ω_2.
c) What resonances can occur in this diagram? How many of them can occur simultaneously?





Problem 3 (8 points, distributed according to (a,b,c) = (4, 1.5, 2.5))
Consider an ensemble of two-level molecules in a host medium. As a result of inhomogeneity in the host, each molecule has a slightly different transition frequency. These frequencies are distributed according to a Gaussian distribution with mean ω_0 and standard deviation D. The coherence between the ground state and the excited state of each molecule decays exponentially with the same homogeneous dephasing rate γ.

a) Explain how one can use the two-pulse echo experiment to measure the homogeneous dephasing rate, even if $D \gg \gamma$. Specifically: describe how the experiment is done and give an expression for the signal. You do not need to give a complete calculation; draw one of the Feynman diagrams that contributes to the two-pulse echo signal and present a clear analysis of the phase factors and relaxation factors and their averages to answer the question.

A more complex echo experiment is the three-pulse echo, in which one uses three pulses ($\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$), and one detects the signal in the direction $\mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}_1$ (so, to lowest order there are three interactions). All pulses have the same frequency; the length of the pulses obeys the same inequalities as in the two-pulse echo, and the time delay between pulse 1 and 2 is τ_1, while the one between pulses 2 and 3 is τ_2. One detects the signal a time t following pulse 3.

b) Show that the three-pulse echo experiment also allows for an echo type signal. To this end draw one Feynman diagram that generates an echo in this experiment.

c) Explain how one may use the three-pulse echo experiment to determine both the homogeneous dephasing rate γ and the population decay rate Γ of the excited state. Like in item a), a full calculation is not necessary.

Problem 4 (6 points, distributed according to (a,b,c)=(2,2,2))
Briefly answer the following questions.

a) What is the wave vector mismatch for sum frequency generation? Argue physically why the wave vector mismatch has an important influence on the signal intensity for sum-frequency generation.

b) What is the slowly varying amplitude approximation in wave propagation? What type of term in the coupled wave equations is neglected in this approximation? (no derivations needed.)

c) What are Rabi oscillations? What determines their frequency? What is the frequency domain signature of Rabi oscillations (no derivations needed).